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human being deal with numbers over 1,000'' is obviously a true proposition, although no instance of such a sum can ever be given. It is therefore perfectly possible to know that there are propositions we do not know, in spite of the fact that we can give no instance of such a proposition. Thus it is fallacious not merely to argue that everything must be known, but even to argue that we can not know that there are things which we do not know. This instance, like many others, illustrates the elementary blunders which philosophers have made owing to their neglect of logic.

To sum up: The primary philosophic effect of the logical doctrines which I share with the "six realists" seems to me mainly negative: it shows that most current philosophical argument is fallacious, and that many questions which have been supposed amenable to a *a priori* treatment must be dealt with empirically, since logic leaves the alternatives undecided. At the same time, in all those matters in which philosophy has been led to contravene science and common sense, there is a presumption, if the arguments of philosophers have been unsound, that their conclusions have been false; hence the logic in question naturally associates itself with pluralism and realism rather than with monism and idealism. Moreover, by the rejection of *a priori* constructions the way is opened for philosophy to become inductive, and to begin the patient cooperative accumulation of results by which the triumphs of science have been achieved.

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REVIEWS AND ABSTRACTS OF LITERATURE

Mysticism in Modern Mathematics. HASTINGS BERKELEY. Oxford University Press. 1910. Pp. 258.

The title of this work will, to many, appear wantonly paradoxical. Mathematics, above all branches of knowledge, is usually credited with the attributes of absolute certainty and of logical precision. Nevertheless, Mr. Berkeley successfully maintains that it contains elements not incor- rectly described by the term mystical. The term requires definition. Mr. Berkeley's meaning will be gathered from the following passage.

"What are we to think of, how shall we characterize, a mental process which might, briefly and in general terms, be indicated thus: Explanation of the derivation, from a primary conception (say that of quantity, or again, of space), of another conception, followed by questions such as these: What is, or what is the nature of, this derived conception? Or—say that Abracadabra is the name of the derived conception—What is the meaning of Abracadabra? The questions imply that the nature of a

derived conception is not made manifest in the account of its derivation, that the meaning of a term may be something other than that which we have agreed to assign to it. Yet, if a derived conception really is present to the mind, to ask what this conception is can only be an indirect way of asking for an explanation of the process by which we have come to form it, that is, of its derivation; and, if we have given the name *Abra-cadabra* to a certain idea, to ask, What is *Abra-cadabra*? is to imply that the meaning of the term is not that idea. But select what term you please to characterize a mental process such as this—I can find none more appropriate than ‘mystical’—and it will probably come to seem inappropriate when you learn further that a notion which is derived, rather than that from which it derives, is the fundamental notion of the subject of thought which involves them both” (p. 64).

To the author, mysticism is a form of illusion, nothing more. In his preface he defines pragmatism as a “methodical and determined attempt to rid philosophy of mysticism.” Throughout the book mystical and mystical illusion are used as synonymous terms. It should be remarked at the outset that this attitude is hardly adequate to the complexity of the problem before us. It is one thing to point out “mystical” tendencies in mathematics; it is quite another to condemn them as mere illusions of symbolism. Where mysticism is to be found, even in the most unlikely places, let us, by all means, recognize it as such; and let us not confuse it with the factual elements with which it is bound up. But the author’s attitude of absolute condemnation requires thorough philosophic substantiation.

The volume is divided into three sections entitled respectively: “Thought and its Symbolic Expression,” “Imaginary Quantities in Algebra and Imaginary Loci in Geometry,” “Metageometry.” The first section is somewhat loosely connected with the remainder and calls for no special comment.

The second section is dominated throughout by the author’s idea of mysticism. In algebra, he deals first with negative quantities and then at considerable length with $\sqrt{-1}$. His conclusions are somewhat vague. He endeavors to show that the geometrical conception of $\sqrt{-1} \times \sqrt{-1}$ becomes *arithmetically* intelligible by stating it in the form $+1: \sqrt{-1}:: \sqrt{-1}: -1$ (pp. 125–6). Undoubtedly this form does illustrate more clearly than the sign of multiplication the relation of perpendicularity. But the arithmetical intelligibility of the process of squaring a quantity which it is not allowable (according to Mr. Berkeley) to isolate, is by no means clear. The problem does not appear to be solved by informing us that “they are then at once recognized as factors or constituent parts of actual expressions, algebraically symbolic, interpreted in terms of generalized number and abstract quantity.” Mr. Berkeley apparently means, though his meaning is obscure, that expressions like $\sqrt{-1} \times \sqrt{-1}$ are similar to dy/dx , which represents an operation. Only conventionally and for specific purposes do we separate dy and dx . Unfortunately for the theory, the sign of multiplication

considered arithmetically does connote the existence of separate quantities to be multiplied, and is not, in any other sense, intelligible. Moreover, their use in algebra depends simply and solely on the convention that they can be treated as separate quantities.

Mr. Berkeley's meaning would have been much clearer if he had dealt with the subject of fractional and negative indices. $a^{\frac{1}{2}}$ and a^{-2} can, in a sense, be called mystical expressions. They have no meaning until we ask, in Mr. Berkeley's words: "What do they mean?" Then we are able to discover for them one interpretation and only one, in this case, purely arithmetical. Why can the mystical and meaningless expression 2^{-2} be used as equivalent to the plain, every-day $\frac{1}{4}$?

The treatment of imaginary points and loci is more explicit, and, whether or no it be philosophically adequate, may assist the elementary teacher, who often finds it difficult to show that this section of analytical geometry represents any form of practical utility. The following quotation will indicate the author's treatment:

"The process, thus far, may be said to exhibit the mathematician at play. To regard it as furnishing a geometrical interpretation of algebraic imaginaries in the usual sense of the word interpretation, or in any sense that would ultimately prompt the question: What is the meaning of an imaginary point? would be to take a *jeu d'esprit* seriously, or to be unconscious or oblivious of the fact that the process is a subtle game in which we play at interpretation, pretend to interpret. If we are conscious of this fact we shall not ask the question; nor shall we suppose that we have found an interpretation in pretending to find one. The game has 'serious scientific value' only if, when we say that the roots of this quadric equation symbolize either real or imaginary abscissæ, we can in this paradoxical phraseology call attention to some geometrical property involved in the system of the circle and the straight line which is independent of their having, in any point or points, coordinates in common, independent, that is, of the straight line being secant or non-secant of the circle."

For purposes of instruction, the exposition will require simplification; but every teacher will appreciate the advantage of being able, where possible, to postpone the consideration of philosophical problems by pointing out that analytical treatment does call attention to new geometrical properties.

The best section in the book is that dealing with metageometry. The author makes an admirable point at the outset by remarking that there is an absurdity, "even a contradiction," in the current term properties of space. He also points out clearly and rightly that the admission of the possible objective existence of non-Euclidean space involves a rejection of our idea of direction. ("—each conception of space must involve a notion of direction peculiar to itself and not comparable with the others.") With Cayley he agrees that the postulate of parallels is self-evident.

Those who uphold this view, however, are logically bound either to assert the self-evidence of the complicated fifth postulate of Euclid (commonly called the twelfth axiom) or to find some satisfactory substitute.

Up to the present the clearest axiom of parallels is that of Playfair. Mr. Berkeley's substitute is worthy of attention. He divides the Euclidean postulate into the following axioms:

"a. Straight lines which are identically inclined to the same straight line are not mutually inclined.

"b. Straight lines which are not mutually inclined are identically inclined to the same straight line."

These should be noted for what they are worth, but it is very doubtful whether they will be considered any simpler, if as simple, as Playfair's axiom.

One other feature of the book it is necessary to mention. The sentences are unnecessarily involved and the whole work is loosely constructed and difficult to read. In many passages the author's meaning is far from clear. For an original essay the quotations are too numerous. As the quotations often convey a more vivid impression than the text, it becomes still more difficult to follow the connection of ideas. Those who, like the technical mathematician, will not agree with the author's main conclusions, may infer confusion of thought from vagueness of expression. The volume would be greatly improved by condensation, by the removal of unessential matter, and by the inclusion of the discussion of other topics (such as fractional and negative indices) which have a direct bearing on the author's line of thought.

Notwithstanding these defects, Mr. Berkeley's essay is worthy of the consideration of the technical mathematician and may be accepted as an interesting addition to the literature on the philosophy of mathematics. Such a volume constitutes further evidence of the growing dissatisfaction with the dogmatism of the mathematicians when they mistake their conceptualisms for objective reality, and when they put forward as probable conclusions which conflict with the dictates of common sense.

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Spinoza's Short Treatise on God, Man and his Well-Being. Translated and edited, with an introduction and commentary and a life of Spinoza.

A. WOLF. London: Adam and Charles Black. 1910. Pp. cxxviii + 246.

As indicated in the title, this work consists of four parts, the life of Spinoza (pp. xi-cii), the history of the "Short Treatise" (pp. ciii-cxxviii), an English translation of the "Short Treatise" (pp. 1-162), and a commentary on the same (pp. 165-240).

The life is admirably done. It is certainly in many respects, probably altogether, the best that has yet appeared in English. The author's birth and religious association have given him certain advantages which he has well utilized in the presentation of the earlier part of Spinoza's life. The account, as a whole, is complete and well told. Little of the source material, as such, has been presented, the author assuming that Freudenthal is sufficiently available to make such presentation useless. Consid-